International Journal of Applied Mathematics &
Statistical Sciences (IJAMSS);
ISSN (P): 2319–3972; ISSN (E): 2319–3980
Vol. 11, Issue 2, Jul–Dec 2022; 1–12

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

ON FUZZY SUB IS-ALGEBRAS

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ABSTRACT

In this paper we study sub IS-, algebra, fuzzy sub IS-, algebra, normal sub IS-algebra, fuzzy normal sub IS-algebra, fuzzy normal sub IS-algebra of fuzzy sub IS-algebra.

***KEYWORDS**: BCI-Algebras, Semigroup, IS-Algebra, Sub IS-Algebra, IS-Algebra Homomorphism, The Cartesian Product, Fuzzy Sub IS-Algebra, Normal Sub IS-Algebra*

Article History

***Received:** 26 Jul 2022* | ***Revised:** 27 Jul 2022* | ***Accepted:** 28 Jul 2022*

1. INTRODUCTION

In 1996, K. Iseki introduced the notion of BCK/BCI- algebras. For the general development of BCK/BCI- algebras [6], In [2]introduced a new class of algebras related to BCI- algebras and semi groups called a BCI- semi group. In this paper we study a new type of fuzzy sub IS-algebra are normal sub IS-algebra, fuzzy normal sub IS-algebra and fuzzy normal sub IS-algebra of fuzzy sub IS-algebra.

2. PRELIMINARY

We review some definitions that will be useful in our results.

Definition 2.1: A Semi group is an ordered pair $(G,\cdot)$,where G is a non-empty set and " ." is an associative binary operation on G. [3]

Definition 2.2 A BCI- algebra is triple (G, *, 0) where G is a non-empty set "*" is binary operation on G, $0 \in G$ is an element such that the following axioms are satisfied for all $s, t, r \in G$:

- $((s * t) * (s * r)) * (r * t) = 0$,
- $(s * (s * t)) * t = 0$,
- $s * s = 0$,
- $s * t = 0$ and $t * s = 0$ implies $= t$

If $0 * s = 0$ for all $s \in G$ then G is called BCK-algebra. [1]

Definition 2.3: An IS-algebra is a non-empty set with two binary operation "*" and "." and constant 0 satisfying the axioms:

- (G,∗, 0) is a BCI-algebra.
- (G, .)is a Semi group,
- s.(t ∗r) = (s.t) ∗(s.r) and (s ∗t).r = (s.r) ∗(t.r), for all s, t, r ∈ *G*. [6]

***Example* 2.4:** let G={0,a,b,c} define "*" operation and multiplication "." by the following tables:

*	0	a	b	c
0	0	0	b	b
a	a	0	c	b
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then by routine calculations we can see that G is an IS-algebra.[6]

***Definition* 2.5:** Let G and Y be IS-algebra a mapping $f : G \to Y$ is called an IS-algebra homomorphism (briefly homomorphism) if $f(x * y) = f(x) * f(y)$ and $f(xy) = f(x) f(y)$ for all $x, y \in G$.

Let $f : G \to Y$ IS-algebra homomorphism. Then the set $\{x \in G : f(x) = 0\}$ is called the kernel of *f*, and denote by *Ker f*. Moreover, the set $\{f(x) \in Y : x \in G\}$ is called the image of f and denote by Im f. [4]

***Definition* 2.6:** Let λ and μ be the fuzzy subsets in a set G, the Cartesian product

$\lambda \times \mu$: G × G ⟶ [0, 1] is defined by $(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}$

for all $x, y \in G$. [9]

***Definition* 2.7:** Let G be a non-empty set a fuzzy subset « of G is a function μ : G→ [0, 1]. [10]

***Definition* 2.8:** Let « and ¬ be a fuzzy sets on G. Define the fuzzy set $« \cap ¬$ as follows: $(« \cap ¬)(x) = \min\{«(x), ¬(x)\}$ for all $x \in G$.[5]

***Definition* 2.9:** Let « and ¬ be a fuzzy sets on G . Define the fuzzy set $« \cup ¬$ as follows:

$(« \cup ¬)(x) = \max\{«(x), ¬(x)\}$ for all $x \in G$.[5]

3. MAIN RESULTS

In this section, we find some results about fuzzy sub IS-algebra, normal sub IS-algebra, fuzzy normal sub IS-algebra and fuzzy normal sub IS-algebra of fuzzy sub IS-algebra.

***Definition* 3.1:** A fuzzy set « defined on G is called a fuzzy sub IS-algebra of G if it satisfies the following conditions:

1) $«(x_1 * x_2) \geq \min\{«(x_1), «(x_2)\}$,
2) $«(x_1 x_2) \geq \min\{«(x_1), «(x_2)\}$ $\forall\, x_1, x_2 \in G$

Proposition 3.2: Let « and ¬ be fuzzy IS-algebra of G. Then $« \cap ¬$ is a fuzzy IS-algebra of G .

Proof: Let « and ¬ are the fuzzy sub IS-algebra and let $x, y \in « \bigcap ¬$ then

$$
\begin{aligned}
(« \cap ¬)(xy) &= \min\{«(xy), ¬((xy)\} \\
&\geq \min\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by \text{ hypothesis}] \\
&= \min\{\min\{«(x), ¬(x)\}, \min\{«(y), ¬(y)\}\} \\
&= \min\{(« \cap ¬)(x), (« \cap ¬)(y)\}.
\end{aligned}
$$

so,

$$
\begin{aligned}
(« \cap ¬)(x * y) &= \min\{«(x * y), ¬((x * y)\} \\
&\geq \min\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by \text{ hypothesis }] \\
&= \min\{\min\{«(x), ¬(x)\}, \min\{«(y), ¬(y)\}\} \\
&= \min\{(« \cap ¬)(x), (« \cap ¬)(y)\}.
\end{aligned}
$$

Hence $« \cap ¬$ is a fuzzy sub IS-algebra.

Proposition 3.3: Let « and ¬ are fuzzy sub IS-algebra of G then $« \bigcup ¬$ is a fuzzy sub IS-algebra of G if $« \subseteq ¬ \;\; or \;\; ¬ \subseteq «$.

Proof: Let « and ¬ are the fuzzy sub IS-algebra, and let $x, y \in « \bigcup ¬$ then

$$
\begin{aligned}
(« \cup ¬)(xy) &= \max\{«(xy), ¬((xy)\} \\
&\geq \max\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by \; hypthoses] \\
&= \min\{\max\{«(x), ¬(x)\}, \max\{«(y), ¬(y)\}\} \quad [« \subseteq ¬ \; or \; ¬ \subseteq «] \\
&= \min\{(« \cup ¬)(x), (« \cup ¬)(y)\}.
\end{aligned}
$$

so,

$$
\begin{aligned}
(« \cup ¬)(x * y) &= \max\{«(x * y), ¬((x * y)\} \\
&\geq \max\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by \; hypthoses] \\
&= \min\{\max\{«(x), ¬(x)\}, \max\{«(y), ¬(y)\}\} \quad [« \subseteq ¬ \; or \; ¬ \subseteq «] \\
&= \min\{(« \cup ¬)(x), (« \cup ¬)(y)\}.
\end{aligned}
$$

Hence $\mu \cup v$ is a fuzzy sub IS-algebra.

Proposition 3.4: Let G be a IS-algebra and let μ, ν, be a fuzzy sub IS-algebra then $\mu \times$ ν is a fuzzy sub IS-algebra of $G \times G$.

Proof: Let « and ¬ are fuzzy IS-algebra ∋ $(x_1, y_1), (x_2, y_2) \in G \times G$ then

$$
\begin{aligned}
(« \times ¬)((x_1, y_1).(x_2, y_2)) &= (« \times ¬)((x_1.x_2, y_1.y_2)) \\
&= \min\{«(x_1.x_2), ¬(y_1.y_2)\} \\
&\geq \min\{\min\{«(x_1), «(x_2)\}, \min\{¬(y_1), ¬(y_2)\}\} \\
&= \min\{\min\{«(x_1), ¬(y_1)\}, \min\{«(x_2), ¬(y_2)\}\} \\
&= \min\{(« \times ¬)(x_1, y_1), (« \times ¬)(x_2, y_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(\mu \times \nu)((x_1, y_1) * (x_2, y_2)) &= (\mu \times \nu)((x_1 * x_2, y_1 * y_2)) \\
&= \min\{\mu(x_1 * x_2), \nu(y_1 * y_2)\} \\
&\geq \min\{\min\{\mu(x_1), \mu(x_2)\}, \min\{\nu(y_1), \nu(y_2)\}\} \\
&= \min\{\min\{\mu(x_1), \nu(y_1)\}, \min\{\mu(x_2), \nu(y_2)\}\} \\
&= \min\{(\mu \times \nu)(x_1, y_1), (\mu \times \nu)(x_2, y_2)\}
\end{aligned}
$$

Hence $\mu \times \nu$ is a fuzzy sub IS-algebra.

***Definition* 3.5:** A fuzzy sub IS-algebra μ of G is said to be normal fuzzy sub IS-algebra if there exists $x \in G$ such that $\mu(x) = 1$.

Remark 3.6: A fuzzy sub IS-algebra μ of G is said to be normal fuzzy sub IS-algebra if and only if μ (0) = 1.

Proof:

Let μ be a normal fuzzy sub IS-algebra of G then

there exists $x \in G$ *such that* $\mu(x) = 1$

since $\mu(0) \geq \mu(x) \quad \forall\, x \in G$

so $\mu(0) \geq 1$ then $\mu(0) = 1$.

Conversely, it is clear.

Proposition 3.7: Let μ and v are normal fuzzy sub IS-algebra of G then $\mu \cap v$ be a normal fuzzy sub IS-algebra of G.

Proof:

Let μ and v are normal fuzzy sub IS-algebra of G then

$\mu \cap v$ is a fuzzy sub IS-algebra of G [by Proposition (3.2)]

also $\mu(0) = 1$ and v (0) = 1 so

$(\mu \cap \nu)(0) = \min\{\mu(0), \nu(0)\} = 1$

therefore $(\mu \cap \nu)$ is a normal fuzzy sub IS-algebra.

Proposition 3.8: Let μ and ν are normal fuzzy sub IS-algebra of G then $\mu \cup \nu$ be a normal fuzzy sub IS-algebra of G if $\mu \subseteq \nu$ *or* $\nu \subseteq \mu$.

Proof:

Let μ and v are normal fuzzy sub IS-algebra of G such that $\mu \subseteq \nu$ *or* $\nu \subseteq \mu$ then

$\mu \cup v$ is a fuzzy sub IS-algebra of G [by Proposition (3.3)]

also $\mu(0) = 1$ and v (0) = 1 so

$(« \cup ¬)(0) = \max\{«(0), ¬(0)\} = 1$

therefore $« \cup \nu$ is a normal fuzzy sub IS-algebra.

Proposition 3.9**:** Let μ and v be a normal fuzzy sub IS-algebra then $« \times ¬$ is a normal fuzzy sub IS-algebra.

Proof:

Let μ and v are normal fuzzy sub IS-algebra of G then,

since μ and v are fuzzy sub IS-algebra

so [by Proposition (3.4)] $« \times ¬$ is a fuzzy sub IS-algebra

Now,

$$(« \times ¬)(0,0) = \min\{«(0), ¬(0)\} = \min\{1,1\} = 1$$

[since μ, v are normal fuzzy sub IS-algebra]

Hence $« \times ¬$ is normal fuzzy sub IS-algebra.

Definition 3.10**:** Let G be a IS-algebra and μ a fuzzy set on X . Then μ is called a fuzzy normal sub IS-algebra of G if it satisfies the following conditions:

1) « is a fuzzy sub IS - algebra of G .
2) $«(x * y) = «(y * x) \quad \forall x, y \in G \setminus \{0\}$ ·
3) $«(xy) = \mu(yx) \quad \forall x, y \in G.$

Proposition 3.11**:** Let μ and v are fuzzy normal sub IS-algebra of G then $« \cap ¬$ be a fuzzy normal sub IS-algebra.

Proof:

Let μ and v are fuzzy normal sub IS-algebra of G,

then $« \cap ¬$ is a fuzzy sub IS-algebra of G [by Proposition (3.2)]

Now,

$$\begin{aligned}(« \cap ¬)(xy) &= \min\{«(xy), ¬(xy)\} \\ &= \min\{«(yx), ¬(yx)\} \; [«, ¬ \; are \; fuzzy \; normal \; sub \text{IS - algebra}] \text{ so,} \\ &= (« \cap ¬)(yx) \;, \quad \forall \; x, y \in G.\end{aligned}$$

$$\begin{aligned}(« \cap ¬)(x * y) &= \min\{«(x * y), ¬(x * y)\} \\ &= \min\{«(y * x), ¬(y * x)\} [«, ¬ \; are \; fuzzy \; normal \; sub \; \text{IS - algebra}] \\ &= (« \cap ¬)(y * x) \quad \forall \; x, y \in G \setminus \{0\}.\end{aligned}$$

therefore $« \cap ¬$ is a fuzzy normal sub IS-algebra.

Proposition 3.12: Let μ and v are fuzzy normal sub IS-algebra of G . Then $\mu \cup \nu$ be a fuzzy normal sub IS-algebra if $\mu \subseteq \nu$ or $\nu \subseteq \mu$.

Proof:

Suppose that μ and v are fuzzy normal sub IS-algebra

then μ and v are fuzzy sub IS-algebra then

$\mu \cup \nu$ be a fuzzy sub IS-algebra [by Proposition (3.3)]

Now,

$$\begin{aligned}(\mu \cup \nu)(xy) &= \max\{\mu(xy), \nu(xy)\} \\ &= \max\{\mu(yx), \nu(yx)\} \qquad [by\ hypothesis\ ] \\ &= (\mu \cup \nu)(yx) \qquad \forall\ x, y \in G.\end{aligned}$$

so,

$$\begin{aligned}(\mu \cup \nu)(x * y) &= \max\{\mu(x * y), \nu(x * y)\} \\ &= \max\{\mu(y * x), \nu(y * x)\} \qquad [by\ hypothesis\ ] \\ &= (\mu \cup \nu)(y * x) \qquad \forall\ x, y \in G \setminus \{0\}.\end{aligned}$$

Hence $\mu \cup \nu$ is a fuzzy normal sub IS-algebra.

Proposition 3.13: Let λ and μ are fuzzy normal sub IS-algebra of G then $\lambda \times \mu$ is a fuzzy normal sub IS-algebra of $G \times G$.

Proof:

Let λ and μ be a fuzzy normal sub IS-algebra of G and let

$(x_1,x_2), (y_1,y_2) \in G \times G$ *where* $x_1,x_2, y_1,y_2 \in G \ni x = (x_1,x_2), y = (y_1,y_2)$

then λ and μ be a fuzzy sub IS-algebra of G so

$\lambda \times \mu$ is a fuzzy sub IS-algebra [by Proposition (3.4)]

now,

$$\begin{aligned}(\lambda \times \mu)(xy) &= (\lambda \times \mu)((x_1, x_2) \cdot (y_1, y_2)) \\ &= (\lambda \times \mu)(x_1 y_1, x_2 y_2) \\ &= \min\{\lambda(x_1 y_1), \mu(x_2 y_2)\} \\ &= \min\{\lambda(y_1 x_1), \mu(y_2 x_2)\} \quad [\lambda, \mu\ are\ fuzzy\ normal\ sub \text{IS-algebra}] \\ &= (\lambda \times \mu)((y_1, y_2) \cdot (x_1, x_2)) \\ &= (\lambda \times \mu)(yx)\end{aligned}$$

and so,

$let\ \ (x_1,x_2),\ (y_1,y_2)\in G\times G\ \ where\ \ x_1,x_2,y_1,y_2\in G\setminus\{0\}$
$such\ that\ \ \ x=(x_1,x_2)\ ,\ y=(y_1,y_2)\in G\times G$

$$
\begin{aligned}
(ª\times«)(x*y) &= (ª\times«)((x_1,x_2)*(y_1,y_2)) \\
&= (ª\times«)(x_1*y_1,x_2*y_2) \\
&= \min\{ª(x_1*y_1),«(x_2*y_2)\} \\
&= \min\{ª(y_1*x_1),«(y_2*x_2)\}\ [ª,«\ are\ fuzzy\ normal\ sub\text{IS-algebras}\ ] \\
&= (ª\times«)((y_1,y_2)*(x_1,x_2)) \\
&= (ª\times«)(y*x)
\end{aligned}
$$

therefore $ª\times«$ is a fuzzy normal sub IS-algebra.

***Proposition 3.14*:** Let G be a IS-algebra and $«$, $ª$ be two fuzzy sets in G such that $«\times ª$ is a fuzzy sub IS-algebra of $G\times G$. Then:

1) either $«(x)\le«(0)$ *or* $ª(x)\le ª(0)$ *for all* $x\in G$.
2) If $«(x)\le«(0)$ *for all* $x\in X$ then either $«(x)\le ª(0)$ *or* $ª(x)\le ª(0)$.
3) If $ª(x)\le ª(0)$ *for all* $x\in X$ then either $«(x)\le«(0)$ *or* $ª(x)\le«(0)$.
4) either $«$ or $ª$ is a fuzzy sub IS-algebra of G .

***Proposition 3.15*:** Let $«\times ª$ be a fuzzy normal sub IS-algebra of G then either $ª$ or $«$ is a fuzzy normal sub IS-algebra of G .

Proof:

Let $«\times ª$ be a fuzzy normal sub IS-algebra of G

so $«\times ª$ be a fuzzy sub IS-algebra of G

then by use Proposition (3.14), either $ª$ or $«$ is a fuzzy sub IS-algebra of G

if $ª$ be a fuzzy sub IS-algebra of G

so [by (3.14)] $ª(x)\le«(0)$

to prove $ª$ is a normal

let $x_1,x_2\in X$ then

$$
\begin{aligned}
ª(x_1x_2) &= \min\{«(0), ª(x_1x_2)\} \\
&= («\times ª)(0, x_1x_2) \\
&= («\times ª)((0,x_1)\cdot(0,x_2)) \\
&= («\times ª)((0,x_2)\cdot(0,x_1)) \\
&= («\times ª)(0, x_2x_1) \\
&= \min\{«(0), ª(x_2x_1)\} \\
&= ª(x_2x_1)
\end{aligned}
$$

Now, let $x_1, x_2 \in G/\{0\}$

$$
\begin{aligned}
ª(x_1 * x_2) &= \min\{«(0), ª(x_1 * x_2)\} \\
&= («\times ª)(0, x_1 * x_2) \\
&= («\times ª)((0,x_1)*(0,x_2)) \\
&= («\times ª)((0,x_2)*(0,x_1)) \quad [«\times ª \; is\, a\; fuzzy\, normal\, sub \text{IS-algebra}\;] \\
&= («\times ª)(0, x_2 * x_1) \\
&= \min\{«(0), ª(x_2 * x_1)\} \\
&= ª(x_2 * x_1) \qquad \forall\, x_1, x_2 \in G\setminus\{0\}\,.
\end{aligned}
$$

Hence $ª$ is a fuzzy normal sub IS-algebra.

In similar way. if $«\times ª$ is a fuzzy normal sub IS-algebra and $«$ is a fuzzy sub IS-algebra.

We can prove that $«$ is a fuzzy normal sub IS-algebra.

***Definition* 3.16:** Let G be a IS-algebra, $«$ and $¬$ are fuzzy sub IS-algebra of G such that $« \subseteq ¬$ then $«$ is called fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$ if :

$$(1) \quad «(y*x) \geq \min\{«(x*y), ¬(y)\}$$

$$(2) \quad «(yx) \geq \min\{«(xy), ¬(y)\} \quad , \; \forall\, x, y \in X\,.$$

***Proposition 3.17*:** Let G be a IS-algebra and let $«$ and $ª$ be fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$. Then $« \cap ª$ is a fuzzy normal sub IS-algebra of $¬$.

Proof:

Let $«$ and $ª$ are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$.

Then $« \cap ª$ is a fuzzy sub IS-algebra [by Proposition (3.2)]

Now, let $x, y \in X$, since

$«(y*x) \geq \min\{«(x*y), ¬(y)\}$, $ª(y*x) \geq \min\{ª(x*y), ¬(y)\}$ and

$«(yx) \geq \min\{«(xy), ¬(y)\}$, $ª(yx) \geq \min\{ª(xy), ¬(y)\}$

therefore

$$1)\quad («\bigcap ª)(yx) = \min\{«(yx), ª(yx)\}$$
$$\geq \min\{\min\{«(xy),¬(y)\},\min\{ª(xy),¬(y)\}\}$$
$$= \min\{\min\{«(xy), ª(xy)\},\min\{¬(y),¬(y)\}$$
$$= \min\{(«\bigcap ª)(xy),¬(y)\}$$

and,

$$2)\quad («\bigcap ª)(y*x) = \min\{«(y*x), ª(y*x)\}$$
$$\geq \min\{\min\{«(x*y),¬(y)\},\min\{ª(x*y),¬(y)\}\}$$
$$= \min\{\min\{«(x*y), ª(x*y)\},\min\{¬(y),¬(y)\}$$
$$= \min\{(«\bigcap ª)(x*y),¬(y)\}$$

Hence $«\bigcap ª$ is a fuzzy normal sub IS-algebra of ¬ .

***Proposition 3.18*:** Let X be a IS-algebra and let « and ª are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra ¬ then $«\bigcap ª$ is a fuzzy normal sub IS-algebra of ¬ if $« \subseteq ª$ or $ª \subseteq «$.

Proof:

Let « and ª are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra¬ .

$«\bigcup¬$ is a fuzzy sub IS-algebra[by Proposition (3.3)]

Now, let $x, y \in G$ then

$$1)\quad («\bigcup ª)(yx) = \max\{«(yx), ª(yx)\}$$
$$\geq \max\{\min\{«(xy),¬(y)\},\min\{ª(xy),¬(y)\}\}$$
$$= \min\{\max\{«(xy), ª(xy)\},\max\{¬(y),¬(y)\}[\sin ce\, « \subseteq ª\, or\, ª \subseteq «]$$
$$= \min\{(«\bigcup ª)(xy),¬(y)\}$$

and so,

$$2)\quad («\bigcup ª)(y*x) = \max\{«(y*x), ª(y*x)\}$$
$$\geq \max\{\min\{«(x*y),¬(y)\},\min\{ª(x*y),¬(y)\}\}$$
$$= \min\{\max\{«(x*y), ª(x*y)\},\max\{¬(y),¬(y)\}[« \subseteq ª\, or\, ª \subseteq «]$$
$$= \min\{(«\bigcup ª)(x*y),¬(y)\}$$

Hence $«\bigcup ª$ is a fuzzy normal sub IS-algebra of ¬ .

***Proposition 3.19*:** If « and ª are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra ¬ then $« \times ª$ is a fuzzy normal sub IS-algebra of $¬ \times ¬$.

Proof:

Let « and ª are fuzzy normal sub IS-algebra of ¬ .

let $(x_1,x_2), (y_1,y_2) \in G \times G$ *such that* $x = (x_1,x_2), y = (y_1,y_2)$

so ¬ , « , ª are fuzzy sub IS-algebra of G,

then ¬ ×¬ is a fuzzy sub IS-algebra [by Proposition (3.9)]

then « × ª is a fuzzy sub IS-algebra of $G \times$ G [by Proposition (3.9)] .

Now, to prove « × ª is a fuzzy normal sub IS-algebra of ¬ ×¬

$$\begin{aligned}
(« \times ª)(yx) &= (« \times ª)((y_1,y_2)(x_1,x_2)) \\
&= (« \times ª)(y_1x_1, y_2x_2) \\
&= \min\{«(y_1x_1), ª(y_2x_2)\} \\
&\geq \min\{\min\{«(x_1y_1), ¬(y_1)\}, \min\{ª(x_2y_2), ¬(y_2)\}\} \\
&= \min\{\min\{«(x_1y_1), ª(x_2y_2)\}, \min\{¬(y_1), ¬(y_2)\}\} \\
&= \min\{(« \times ª)((x_1,x_2)(y_1,y_2)), ¬ \times ¬(y_1,y_2)\} \\
&= \min\{(« \times ª)(xy), ¬ \times ¬(y)\}
\end{aligned}$$

and so,

$$\begin{aligned}
(« \times ª)(y * x) &= (« \times ª)((y_1,y_2) * (x_1,x_2)) \\
&= (« \times ª)(y_1 * x_1, y_2 * x_2) \\
&= \min\{«(y_1 * x_1), ª(y_2 * x_2)\} \\
&\geq \min\{\min\{«(x_1 * y_1), ¬(y_1)\}, \min\{ª(x_2 * y_2), ¬(y_2)\}\} \\
&= \min\{\min\{«(x_1 * y_1), ª(x_2 * y_2)\}, \min\{¬(y_1), ¬(y_2)\}\} \\
&= \min\{(« \times ª)((x_1,x_2) * (y_1,y_2)), ¬ \times ¬(y_1,y_2)\} \\
&= \min\{(« \times ª)(x * y), ¬ \times ¬(y)\}
\end{aligned}$$

Hence « × ª is a fuzzy normal sub IS-algebra of¬ ×¬ .

***Proposition 3.20*:** Let $f : G \to$ Y be a homomorphism if « is a normal fuzzy sub IS-algebra of Y then $«^f$ is a normal fuzzy sub IS-algebra of G.

***Proposition 3.21*:** Let $f : G \to$ Y be a homomorphism if « is a fuzzy normal sub IS-algebra of a fuzzy sub IS-algebra¬ . Then $«^f$ is a fuzzy normal sub IS-algebra of $¬^f$.

Proof:

Let « is a fuzzy normal sub IS-algebra of ¬ . Then

$$«^f(x) = «(f(x)) \leq ¬(f(x)) = ¬^f(x) \qquad [\sin ce\ « \subseteq ¬].$$

and $«^f$ is a fuzzy sub IS-algebra[by Proposition (3.20)]

$¬^f$ is a fuzzy sub IS-algebra

Now, to prove $«^f$ is a fuzzy normal sub IS-algebra of $¬^f$ thus

$$
\begin{aligned}
«^{f}(yx) &= «(f(yx)) \\
&= «(f(y)f(x)) \\
&\geq \min\{«(f(x)f(y)), ¬(f(y))\} \\
&= \min\{«(f(xy)), ¬^{f}(y)\} \\
&= \min\{«^{f}(xy), ¬^{f}(y)\}
\end{aligned}
$$

so,

$$
\begin{aligned}
«^{f}(y*x) &= «(f(y*x)) \\
&= «(f(y)*f(x)) \\
&\geq \min\{«(f(x)*f(y)), ¬(f(y))\} \\
&= \min\{«(f(x*y)), ¬^{f}(y)\} \\
&= \min\{«^{f}(x*y), ¬^{f}(y)\}
\end{aligned}
$$

Hence $«^{f}$ is a fuzzy normal sub IS-algebra of $¬^{f}$.

Proposition 3.22: Let $f : G \rightarrow \mathrm{Y}$ be epimorphism if $«^{f}$ is a normal fuzzy sub IS-algebra of G then $«$ is a normal fuzzy sub IS-algebra of Y.

Proposition 3.23: Let $f : G \rightarrow \mathrm{Y}$ epimorphism if $«^{f}$ is a fuzzy normal sub IS-algebra of $¬^{f}$. Then $«$ is a fuzzy normal sub IS-algebra of V.

Proof:

Let $«^{f}$ is a fuzzy normal sub IS-algebra of $¬^{f}$ then

since *f* is an epimorphism if $x \in \mathrm{Y}$ $\exists\, a \in \mathrm{X}$ *such that* $f(a) = x$

$$«(x) = «(f(a)) = «^{f}(a) \leq ¬^{f}(a) = ¬(f(a)) = ¬(x) \;\; so \;\; «(x) \subseteq ¬(x) \;\; \forall x \in \mathrm{Y} .$$

and $«$ is a fuzzy sub IS-algebra[by Proposition (3.22)]

Now, let $x, y \in \mathrm{Y}$ $\exists\, a, b \in G$ *such that* $f(a) = x$, $f(b) = y$ then

$$
\begin{aligned}
«(yx) &= «(f(b)f(a)) = «(f(ba)) = «^{f}(ba) \\
&\geq \min\{«^{f}(ab), ¬^{f}(b)\} \\
&= \min\{«(f(ab)), ¬(f(b))\} \\
&= \min\{«(f(a)f(b)), ¬(y)\} \\
&= \min\{«(xy), ¬(y)\}
\end{aligned}
$$

and so,

$$
\begin{aligned}
«(y*x) &= «(f(b)*f(a)) \\
&= «(f(b*a)) \\
&= «^{f}(b*a) \\
&\geq \min\{«^{f}(a*b), ¬^{f}(b)\} \\
&= \min\{«(f(a*b)), ¬(f(b))\} \\
&= \min\{«(f(a)*f(b)), ¬(y)\} \\
&= \min\{«(x*y), ¬(y)\}
\end{aligned}
$$

Hence « is a fuzzy normal sub IS-algebra of ¬ .

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